EEC289U: The Fourier Transform and Its Applications in Imaging

CRN: 54031

Units: 4 (Lecture/Discussion)

Prerequisites: Junior/Senior or Graduate standing. EEC130A and EEC150, or equivalent courses, or instructor consent.

Meeting times: TR 3:10-4:30pm, Olson Hall 159

Grading: Weekly homework (30%), Midterm exam (35%), Final exam (35%)

Catalog Description: A broad and deep understanding of the techniques and principles of Fourier analysis, first in one dimension, and then apply them to the analysis of two-dimensional imaging systems.

Expanded Course Description:

There is an ever-increasing interest in the techniques, tools and technology in imaging. Important fields such as biology, medicine, autonomous vehicles, satellite reconnaissance, smart phones and astronomy, to name a few, rely heavily on the new imaging modalities offered by modern technology. At the core of these new technologies are the fundamental principles contained in the frequency- domain analysis afforded by two-dimensional Fourier transform theory. The purpose of this course is to provide a broad and deep understanding of the techniques and principles of Fourier analysis, first in one dimension, and then apply them to the analysis of two-dimensional imaging systems. This will give students an in-depth exposure to the principles and applications of the Fourier transform in one and two dimensions in both space and time and then apply them to standard and modern imaging systems. These will include simple diffraction theory from apertures and objects, gratings, imaging, spatial filtering, radiotelescope arrays and, depending on interest and available time, holography and/or computed tomography.

Apart from the emphasis on imaging, the student will gain a good understanding of the powerful techniques that use the tools of Fourier analysis and this will, in turn, prepare them for any of the myriad courses in which Fourier analysis is used. Such courses might include:

- Analog and digital signal processing.
- Communications theory (modulation, noise, etc).
- Solid state physics.
- Optics.
- Astronomy.
- Biomedical imaging.

Suggested Reading: Course notes related to the lectures and recommended supplemental texts *“The Fourier Transform and its Applications”* by R. N. Bracewell (McGraw-Hill) and *Fourier Optics* by J. W. Goodman.

Course Outline:

Week #1.

1. The Trigonometric Fourier Series. Orthogonal bases. Wave functions and arguments.
2. The Complex Fourier Series. 3D perspective plots of complex Fourier series spectra.
3. Transition from the Fourier series to the Fourier transform.
4. Alternative forms of the Fourier transform and its inverse.
5. Compact notations.
6. Transform pairs and theorems to memorize.

Week #2.

1. Validity of the Fourier transform.
2. Existence conditions for the Fourier transform.
3. The generalized Fourier transform for handling functions which violate the existence conditions.
4. Functions defined in terms of the limit of a sequence.
5. Definitions for the delta function.
6. Properties of delta functions: unit area, the sifting property, scaling law, derivatives.

Week #3.

1. The cyclic property of the Fourier transform.
2. Fourier transform theorems: linearity, scaling, shift, Parseval.
3. Convolution. Mathematical definition and pictorial exposition.
4. Properties of the convolution integral (commutative, associative, distributive over addition).
5. Convolution and the impulse response.
6. Convolution with the delta function: replication.
7. The autocorrelation and autoconvolution functions.

Week #4.

1. More Fourier transform theorems: convolution, modulation, autocorrelation.
2. The sampling function. Time and frequency domain descriptions.
3. Symmetry properties of the Fourier transform.

Week #5.

1. Two-Dimensional Fourier Transformation and Linear Systems.
2. Two-dimensional frequency spectra.
3. Functions that are separable in two dimensions and their transforms.
4. Two-dimensional delta functions.
5. Periodic surfaces and their spatial frequency spectra.
6. Functions with circular symmetry: Fourier-Bessel (Hankel) transforms.

7. Frequently-used two-dimensional functions

Week #6.

1. Linear Systems and the Two-Dimensional Impulse Response.
2. Two-dimensional impulse response and the point spread function.
3. Time- and space-invariance: superposition ⇒ convolution.
4. Optical systems: *Object plane* + *Linear space-invariant system* + *Image plane*.
5. The convolution theorem in two dimensions: the transfer function.
6. Sampling in two dimensions.

Week #7.

1. Review of basic principles of wave propagation.
2. Infinite transverse plane waves.
3. Decomposition of wavevectors, direction cosines.
4. Spherical waves and offset source points.
5. The quadratic approximation to spherical wavefronts.
6. Propagation of the angular spectrum.
7. The effects of a diffracting aperture on the angular spectrum.
8. Diffraction limits: optical telescopes, radio telescopes, human eye.
9. Periodic amplitude and phase perturbations within apertures: diffraction gratings.

Week #8.

1. Introduction to Scalar Diffraction Theory
2. Fresnel diffraction and the Cornu spiral.
3. Fraunhofer diffraction.
4. The lens as a spatial phase transformation.
5. Paraxial approximation ⇒ quadratic phase transformation.
6. Focal length, converging and diverging lenses.
7. Fourier transforming properties of lenses.
8. Effect of finite aperture: vignetting.

Week #9.

1. Impulse response of a positive lens.
2. Relation between object and image. Image inversion. Magnification.
3. Entrance and Exit Pupils.
4. Monochromatic illumination: coherent vs. incoherent.
5. Spatial vs. temporal coherence.
6. Coherent transfer function, optical transfer function, modulation transfer function.
7. MTF of common lenses, USAF test target, optimum aperture.

Week #10.

1. Frequency Analysis of Optical Imaging Systems
2. Spatial Filtering and Optical Information Processing
3. Analysis $\Rightarrow$ Synthesis.
4. Applications: character recognition, information processing, image restoration.
5. Abbe-Porter experiment.
6. Zernike phase-contrast microscope.
7. Schlieren photography.
8. Vander Lugt filter and character recognition.
9. Holography.
10. Time Permitting:
    a. Radiotelescopes and aperture synthesis.
    b. Computed Tomography:
    c. Projections, line integrals and the Abel transform.
    d. Cyclic relationships: Abel, Fourier and Hankel transforms.
    e. The Projection-Slice Theorem.
    f. Computed tomography and the general projection-reconstruction problem.